Mitigating Non-Strategic Coalitions

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A common problem with three-player games is 'kingmaking', in which a player with no hope of winning is able to determine the eventual winner. I describe a known method for mitigating this problem and its modification for games that include final ranks. I also introduce the related term 'princemaking' to describe cases in which the leading player is able to determine the second place-getter, and strategies for mitigating this problem as well.

1 Introduction

COMBINATORIAL games are two-player games with no hidden information and no chance elements [1]. Games between more than two players are susceptible to *non-strategic coalitions* [2] in which players may pursue personal agendas rather than playing strictly to win. A well known example is the *kingmaking* problem, in which a player with no hope of winning is able to determine the eventual winner [3, 4], which can ruin a game for many players [5].

A lesser known type of non-strategic coalition problem is what I call *princemaking*, which occurs when the *leading* player is able to determine who comes second. This article explores the issues of kingmaking and princemaking and presents ways to mitigate these effects. Future references to 'game' in this article refer to combinatorial games.

2 The Kingmaking Problem

An interesting feature of three-player games is emergent temporary strategic alliances between two players, typically to thwart the current leader. Algorithms have been developed in an attempt to understand these social dynamics, but formal studies of these, such as [6], remain inconclusive, making it hard to say at a formal level what is 'rational' play in such games.

However, the intriguing social features of three-player games also create the potential for kingmaking. This is a problem for many players, as the chosen winner may consider the victory to be hollow, and the remaining player may resent having the chance of victory snatched away by the kingmaker rather then through their own strategic errors.

The very features of abstract strategy games that appeal to their devotees, namely the significant player control due to no hidden information or randomness, also exacerbate kingmaking in three-player games. A kingmaker has full knowledge of the game state (no hidden information) and more ability to manipulate the game (no randomness).

3 The Stop-Next Rule

One way to mitigate kingmaking is the Stop-Next (SN) rule:

> *Players may not let the next player win on the next turn, unless there is no other choice.*

This rule was developed in 2002 to specifically address the problem of kingmaking in three-player games [7, pp. 161–165].

3.1 Yavalath

A well-known game featuring SN is Yavalath from 2007 [8, pp. 75–86].

Yavalath is played on a hexagonal grid of hexagons with five cells per side.

The board starts empty. Players take turns adding a piece of their colour to an empty cell.

Players win by making a line of four (or more) of their pieces, but lose by making a line of exactly three beforehand.

The game is tied if the board fills up before any player wins.

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Yavalath was originally designed for two players, but later extended to three players by adding the following rules [8, pp. 75–86]:

A player must block the next player's win if possible.

Losing players leave the game but their pieces remain on the board.

The winner is either the last surviving player or the first player to form a line of four (or more) of their pieces.

While Yavalath is most famous for having been designed by computer [9], it is also known among board gamers for working well with three players. A surprising amount of drama and strategy emerges from this simple rule set, and SN is an integral part of making the three-player version work. To appreciate why, consider Figure 1, which shows a game with turn order White/Black/Red with Black to move.

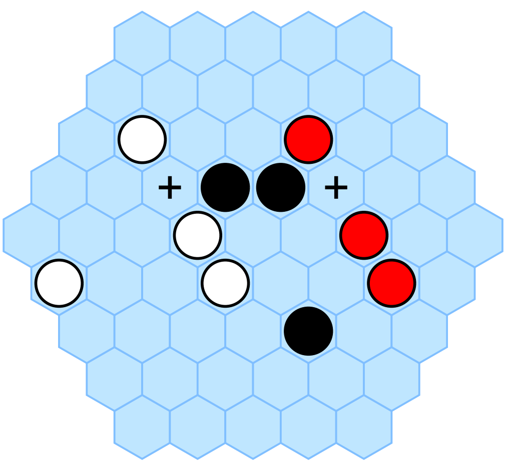


Figure 1. Black to move in Yavalath.

Both White and Red threaten to win next turn with a line of four (at the points marked +). If SN were not in place, then Black could decide who wins this game despite having no chance of winning themselves. If Black blocks the red line, then White will win the game (the triangle on the left gives White a winning advantage). Alternatively, if Black plays anywhere else then Red will complete a line of four to win.

The SN rule *requires* Black to black the red line, giving Black no choice in the matter. Note that SN is an exploitable strategy in its own right, which can be used to manipulate opponents into making unfavourable moves; the key difference being that these moves will be dictated predictably by the rules of the game and not the opponent's personal agenda. The next game, Chromatix,[1] shows such manipulation in action.

3.2 Chromatix

Chromatix is played by Pink, Grey and Maroon on a hexhex board with opposite sides bearing their colour. Players own stones that mix to form their colour:

- Pink owns red and white stones.
- Grey owns black and white stones.
- Maroon owns red and black stones.

Players take turns placing one of their stones on any empty cell.

A player wins by connecting their two opposite sides by a group of stones of their own colours. SN is in effect.

In an advanced version, a player can also win by connecting three non-adjacent sides.

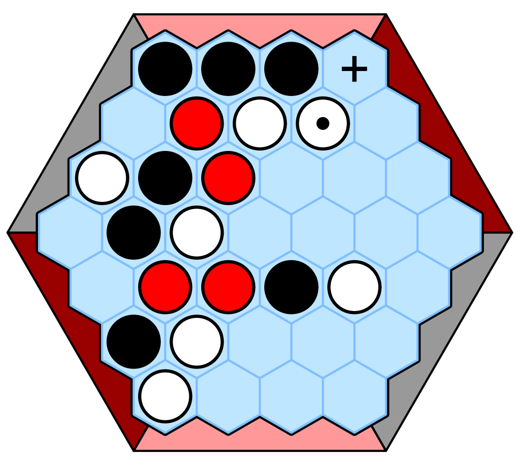


Figure 2. Grey's turn in Chromatix.

Figure 2 shows a Chromatix game in progress. For black and white readers, Pink wants to connect the N and S sides, Grey

[1] https://www.boardgamegeek.com/boardgame/178059/chromatix

the SE and NW sides, and Maroon the SW and NE sides. Pink has just placed the white piece marked with a dot, making it Grey's turn to move (the order of play is shown in the rules).

Grey has no winning move, but Maroon is one move away from winning, since a red or black stone at **+** will create a red/black connection between the maroon sides.[2] SN requires Grey to stop Maroon from winning next turn. Grey can do this only by playing a white stone at **+**, as in Figure 3.

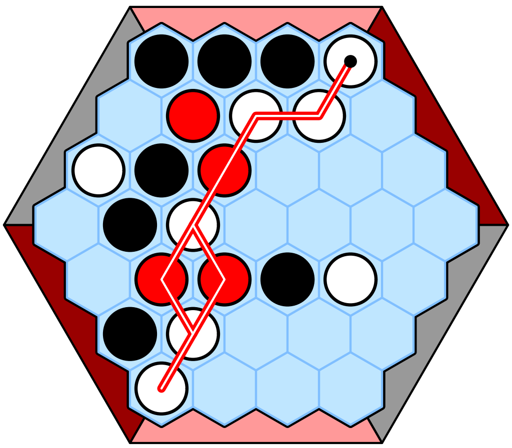


Figure 3. Grey is forced to block a Maroon win, which achieves a Pink win.

This move, however, creates a red/white connection between the pink sides. Thus SN forces Grey to give Pink the win on Grey's turn. Pink was able to bring the game to a state in which both opponents were one move away from winning, knowing that SN would require Grey to give Pink the win.

Thus SN can mitigate kingmaking in three-player abstract strategy games and at the same time add strategic interest. However, 'mitigate' is not 'eliminate'; it is still legal for a player to intentionally play to set up the second player for a win on a later turn (beyond the next), and it is legal for a player to intentionally play in a way that sets up the third player for a win on the third player's very next turn.

3.3 McCarthy's Revenge Rule

An alternative to SN for mitigating kingmaking is provided by McCarthy's Revenge Rule. This fairly well-known rule was proposed around 1950 by John McCarthy during gaming sessions with John Nash, Lloyd Shapley and Martin Shubik [10, p. 390]:

> *If you can not win the game, then hurt the player who has hurt you the most.*

While this revenge rule has psychological appeal, it may be unclear who is actually to blame for your predicament. The exact perpetrator may be hard to determine amidst the game's complexity, or multiple opponents may be equally responsible, or perhaps a player only has themself to blame for poor play.

Another practical problem is that of enforcement. It may be unclear what your best moves are to hurt your victim, and other players may disagree whether you are being appropriately vengeful. Further, Browne has recently demonstrated that hurting the opponent who has hurt you the *least* may be more effective in some cases [2].

This rule can be hard to enforce in practice, so is perhaps best left as a form of etiquette between players. SN has the advantage of mitigating the kingmaker problem in a transparent and unambiguous way.

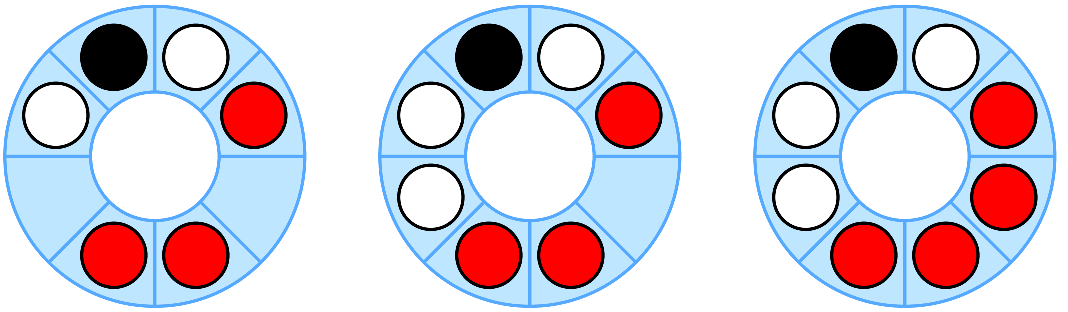
4 Generalising Stop-Next

SN has a side effect which may be undesirable for some players in three-player games with *final ranks*, i.e., first place, second place, third place. It obliges the current player to prevent the next player's immediate win, even when doing so worsens the current player's own final rank. I present a version of SN which not only prevents non-strategically harming others, but also does not require non-strategically harming oneself.

4.1 Alaric

The following example from Bill Taylor's game Alaric, which itself is a variant of Alak [11], demonstrates this problem with final ranks.

[2] Note that corner hexes belong to both of the adjacent sides which they touch.



a *b* *c*

Figure 4. An example Alaric endgame with White to play next.

Alaric is played on a circular ring of cells by White, Red and Black. Players takes turns placing a stone of their own colour on an empty cell.

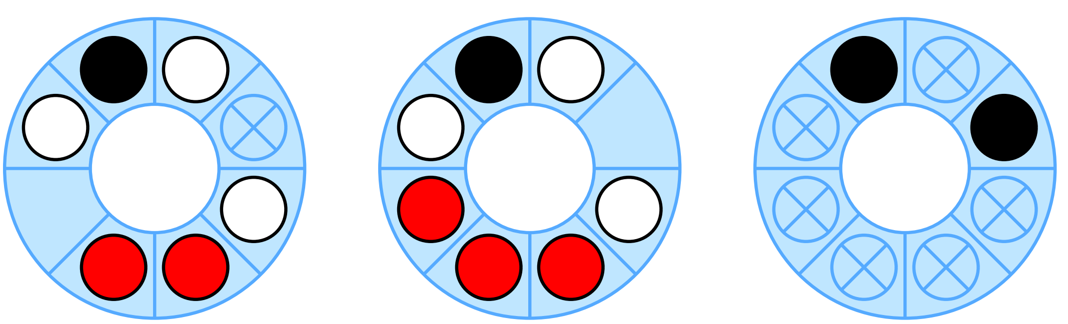
If the move causes a consecutive run of enemy stones (of either colour) with no adjacent empty cell(s), then those stones are captured and removed. The next player may not play onto any of the newly empty cells.

The game ends when the next player has no legal move. It is won by the player with the most *territory*, i.e. the number of that player's stones in play plus the number of empty cells in continuous runs that touch only that player's stones.

Board *a* in Figure 4 shows a sample Alaric endgame, with White to play next. If White plays on the empty cell on the left, then the result is board *b*. Now Red's only move is to play on the remaining open cell, resulting in board *c*. Black has no legal move, so the game formally finishes at the end of Red's turn. The final scores are 4/3/1 making Red the winner. White's move *a* therefore led to an immediate win for the next player (Red).

With SN in effect, White must play instead on the empty cell on the right to capture the single red piece, resulting in board *d* in Figure 5. The newly vacated cell (marked $\times$) is unplayable for the next player (Red), whose only option is to make move *e*. The blocked cell is now playable again, so Black plays there, capturing all opponent stones (board *f*). There are no legal spaces where White can play, so the game ends with a clean 8/0/0 sweep by Black.

Thus without SN, White can achieve second place ahead of Black in third place, but with SN, White is forced to tie with Red. The following modification is proposed to avoid this problem.



d *e* *f*

Figure 5. Alternative play by White.

4.2 General-Stop-Next

I propose the following general principle:

> *Players must block a win by the next player* unless *such a block causes self-harm.*

Formalising this idea into a precise rule proved tricky, but Bill Taylor, João Neto and I eventually hit upon the following rule that I call General-Stop-Next (GSN):

> *The mover may not choose any move that gives the next player an immediate win, unless the mover's only other moves would let the third player win on their next turn (regardless of the next player's possible moves) in a way that worsens the mover's own final rank.*

GSN prevents self-harm in cases such as the Alaric example shown, but does not prevent all cases of self-harm. For instance, consider a case in which the third player does not have an immediate win next turn, but in which the mover will provably finish in a lower final rank by stopping the next player; GSN still requires the mover to stop the next player and accept the lower rank.

A more general rule that permits the mover to let the next player win whenever the mover can prove that stopping the next player will ultimately reduce the mover's final rank would surely be impractical, requiring confusing proofs about future continuations, and probably resulting in disagreement among players. So a more permissive rule of this sort would potentially mire the game in intractable complexities of adjudication. The more restricted formulation of GSN, which requires looking ahead only two moves, is complicated enough. Note that GSN is equivalent to SN in games with no final ranks.

5 Princemaking Problem

Just as a player whose loss is certain may find themself in a position to determine which opponent wins, a player whose win is certain may find themself in a position to determine which opponent gains second place. This problem, which I call 'princemaking', is undesirable for the same reasons as kingmaking.

5.1 Frozen-Irrelevance

One possible way to mitigate princemaking is to include a 'catch-up mechanism' to prevent a runaway leader in the first place.[3] Another possible way (which can be combined with a catch-up mechanism) is to not use final ranks at all: simply end the game when it is provably certain that one player has won, and do not distinguish second and third place. A (less drastic) third option is to remove the certain winner from further play in the game, but let the two remaining opponents continue to battle for second place. This latter option, dubbed *Frozen-Irrelevance* (FI) by Bill Taylor, works as follows:

> *A player guaranteed of victory is frozen from further play, making that player irrelevant to the battle for second place.*

This is similar in principle to the mechanism described by P. D. Straffin, Jr. for Three-Player Hex [10, p. 393], in which a player with no way to win is eliminated from the game. This suggests a more general form of FI in which a player is frozen as soon as a given rank is provably achieved – be it first, second or third – which could have potential benefits for both kingmaking and princemaking. However, I focus on the more specific formulation above to address princemaking in particular here.

5.2 Feedback Morro

Feedback Morro is a three-player game that uses FI and a catch-up mechanism to mitigate the problem of princemaking. It was developed by João Neto, Bill Taylor and me in 2015 as a three-player variant of Morro.[4] Figure 6 shows a game of Feedback Morro after each player has played $n-1$ turns.

[3] Catch-up mechanisms also help reduce kingmaking by making it harder for players to fall hopelessly behind.
[4] https://www.boardgamegeek.com/boardgame/127628/morro.

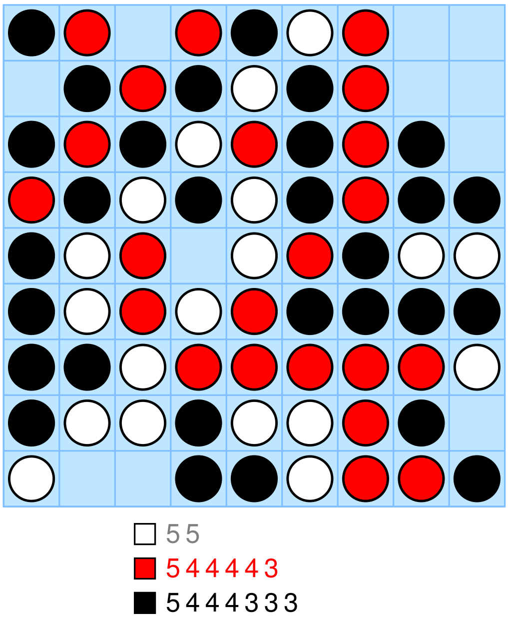


Figure 6. Feedback Morro after round $n-1$.

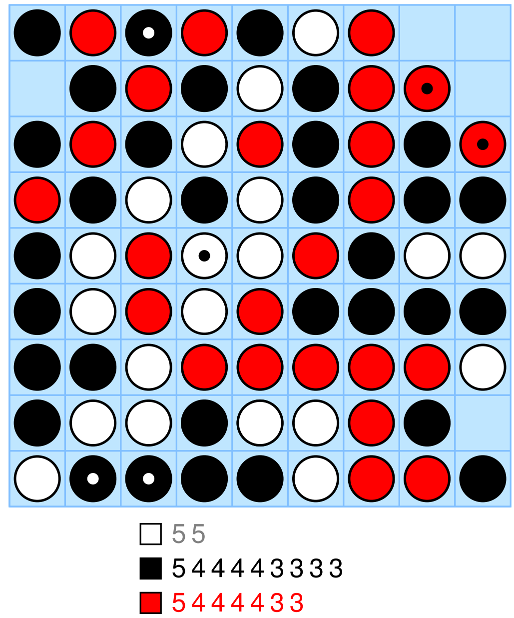


Figure 7. Feedback Morro after round n.

Feedback Morro is played on a 9×9 grid by White, Red and Black. Players in turn place one or more of their stones on empty cells.

Players try to form lines of their own stones, either horizontally, vertically or diagonally. Lines of the same length *match*. The player with the longest *unmatched* line at the end of the game wins.

The number of stones that players place each turn is equal to their current rank at the start of their turn:

- First place plays one stone.
- Second place plays two stones.
- Third place plays three stones.

FI: A player who creates an unbeatable line immediately wins and is frozen from further play, while the remaining players continue to battle for second place.

Each player's current score information is shown below the board, with the players listed in order of score. White is currently in first place having two 5-lines (i.e. lines of length 5), while the opponents each have only one 5-line. Red has four 4-lines compared to Black's three 4-lines, so Red takes second place.

In the next game round n, White will play one stone, Red (who will still be second) will play two stones, and Black (who will still be third) will play three stones. Suppose that White, Red and Black then play the marked stones in Figure 7. The table below the board shows the updated player ranks.

White's single stone blocks Red's diagonal 4-line from extending. This was the last remaining chance for any of White's opponents to make a second 5-line. White's current two 5-lines can therefore not be beaten – or even matched – making White the winner.

With FI in place, White must stop playing while Red and Black continue to play on for second place. Note that Black made another 4-line, so Black and Red are tied in the number of 4-lines. Black has more 3-lines than Red, so Black has advanced to second place, while Red has fallen to third. Thus, Red will play three stones in round $n+1$ and Black will play two stones (if Red's move does not change the rankings).

5.2.1 Continuation with FI

How will round $n+1$ proceed with rational play from the remaining players? Neither Red nor Black can create any more 4-lines in round $n+1$, so second place will be decided by shorter lines. Figure 8 shows plausible moves by Red and Black in round $n+1$ (White has been frozen so did not move).

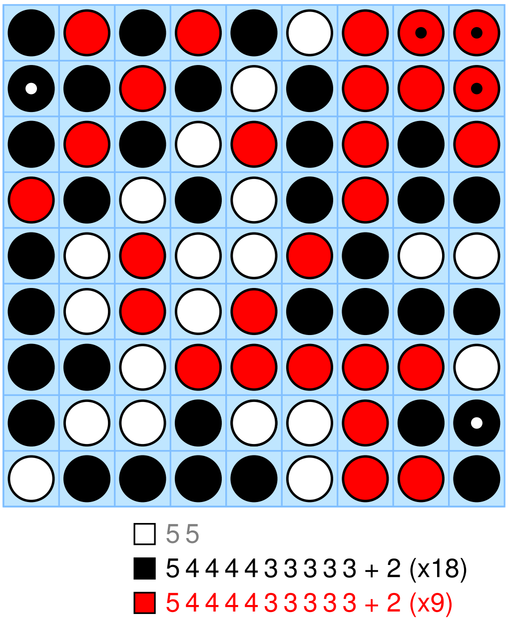


Figure 8. Game after round $n+1$ (with FI).

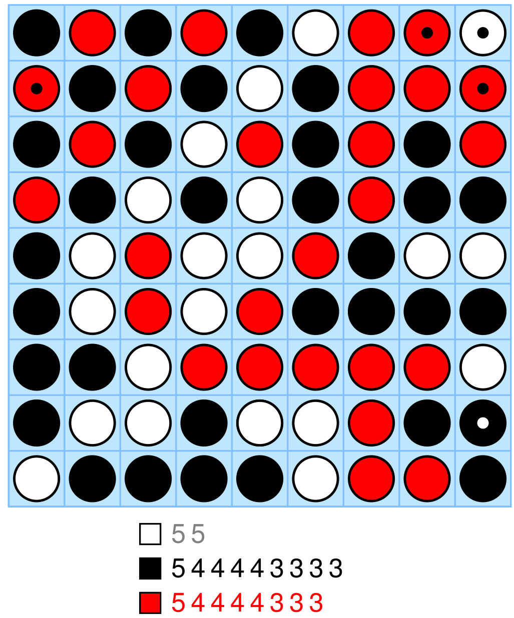


Figure 9. Without FI: Black comes second.

Red created three more 3-lines by playing in the upper right. Black was entitled to three stones (since Red's new 3-lines demoted Black to third place), but could only play in the two remaining empty cells, creating one more 3-line in the upper left. Black and Red are tied for 5-lines, 4-lines and 3-lines, but Black has more 2-lines, giving Black second place. The result with FI in place is: White, Black, then Red.

5.2.2 Continuations without FI

Let us consider the same game without FI in force. In this case, White will continue to play in round $n+1$, despite having an invulnerable lead after turn n, and White's move will determine which opponent finishes in second place.

White can engineer second place for Black by playing in the top right cell. Red can then make only one more 3-line, as shown in Figure 9, allowing Black to retain second place.

Alternatively, White can engineer second place for Red by playing in the upper left cell. This blocks Black from making any more 3-lines, as shown in Figure 10, giving Red second place. This example demonstrates how the FI rule can act to prevent the winning player from choosing who comes second.

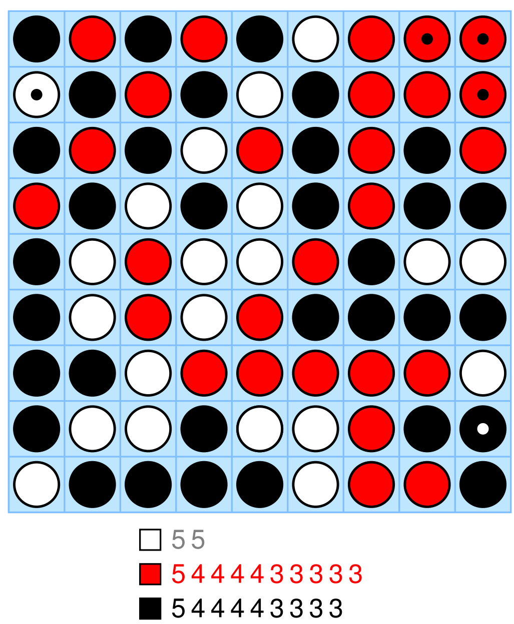


Figure 10. Without FI: Red comes second.

6 Conclusion

Abstract strategy games with three players enable interesting mechanisms not found in two-player games, such as temporary alliances, but also enable detrimental side-effects such as kingmaking and princemaking. Kingmaking can be mitigated by using a simple Stop-Next rule, which can be generalised to also work for three-player games in which final positions are ranked.

Princemaking can be mitigated by catch-up mechanisms to keep scores close, and by the Frozen-Irrelevance rule, which requires a player with an invulnerable lead to cease further play. With these two means of mitigating the effects of non-strategic coalitions, many three-player abstract strategy games may become more viable and interesting games. This is good news for designers and players of abstract games; the design space of three-player abstracts is relatively unexplored compared to that of two-player games.

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