

# 2024 IC Math Exploration Day Team Competition

1. There is one negative integer whose square is 30 more than itself; which one?

2. Solve for  $x$ :  $\log(\log(x)) - \log(\log(7)) = \log(2)$ .

3. If it takes 6 students 9 hours to decorate the Math Department, how long will it take 10 students to complete the task?

4. The mean and median of a list of 5 positive integers are both 7. What is the largest possible number in the list?

5. What is the area of the smallest circle that can be circumscribed about a square of area 16?

6. 1 is the first odd natural number, 3 is the second, and so on. What is the 2024<sup>th</sup> odd natural number?

7. Define the operation  $\star$  by  $x \star y = x^3 - y^3$ . What is the value of  $(1 \star -2) \star (1 \star 2)$

8. You play a best of seven series of Monopoly games with an evenly-matched friend. This means that the first one to win four games is the champion. What is the probability that all seven games are required?

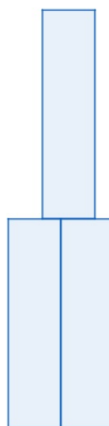
9. Teresa and Megan buy tickets to a play. Teresa buys six tickets using a coupon that gives her a 25% discount on each ticket and Megan buys four tickets using a coupon that gives her a 15% discount on each ticket. If Teresa paid a total of \$88 more than Megan, what is the price of each ticket?

10. If  $\tan(x) = 3/4$ , then what is the value of  $\csc(x) + \sec(x)$ ? Express the answer in the form  $a/b$ .

11. Joash filled a 550 gallon dunk tank for the county fair. Pete filled a child's play pool, using a hose with one-half the fill rate of Joash's hose. If it took Pete one-fifth the time to fill the pool that it took Joash to fill the tank, how much water does the play pool contain?

12. Given two numbers  $a$  and  $b$ , their average is given by  $\frac{a+b}{2}$  and their harmonic mean is given by  $\frac{2}{\frac{1}{a} + \frac{1}{b}}$ . If the average of two numbers is 64 and their harmonic mean is 60, then what is the product of those two numbers?

13. The following shape is made up of three congruent rectangles, each with perimeter 37 cm, touching along some sides. What is the perimeter of the shape?



14. Our classroom has a large jar of candies of different colors. The probability of choosing each color is given in the following table:

color	red	orange	yellow	green	blue
probability	0.13	0.30	0.17	0.15	

If you were to pick 40 candies from the jar, how many blue candies would you expect to pick?

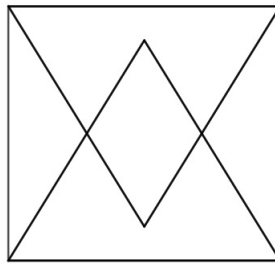
15. What is the product of the largest two-digit prime number and the smallest three-digit prime number?

16. Consider the sequence  $a_1 = 4$ ,  $a_2 = 6$ , and for  $n > 2$ , we have  $a_n = \frac{a_{n-1}}{a_{n-2}}$ . What is the value of  $a_{2024}$ ?

17. Consider the curve given by the equation  $36x^2 + 16y^2 = 576$ . Let  $x_{\max}$  be the largest  $x$ -coordinate of a point on the curve and let  $y_{\max}$  be the largest  $y$ -coordinate of a point on the curve. What is the value of  $x_{\max} + y_{\max}$ ?

18. If  $S_1 = 20 + 22 + 24 + \cdots + 400$  and  $S_2 = 22 + 24 + 26 + \cdots + 402$ , then what is the value of  $S_2 - S_1$ ?

19. Two equilateral triangles have bases that are the opposite edges of a unit square, as in the image below. What is the area of overlap of the two triangles?



20. Given a positive integer  $n$ , we define  $s(n)$  to be the sum of the digits of  $n$ . For how many values of  $n$  does  $n + s(n) = 2024$ ?